

Supplemental Material: SZ Sequences: Binary-Constructed $(0, 2^q)$ -Sequences

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S-1 ADDITIONAL RESULTS

S-1.1 Rendering

Fig. S1 presents additional results for the *Sportscar* scene with additional environment maps to supplement Fig. 5. Together, we have the following environment maps, spanning a range of illumination:

- *Empty Warehouse*: the interior of a large warehouse, primarily lit by fluorescent lights but with a small window allowing some daylight. (Fig. 5).
- *Cayley Interior*: the interior of a house, mostly illuminated with daylight through the doors of a balcony.
- *Hangar Interior*: the interior of an airplane hangar, lit by daylight both through skylights and a large open door.
- *Sky*: an analytic sky model [Hošek and Wilkie 2012, 2013].

For all of these (and for Fig. 5), we report the average error over 50 images rendered with different random seeds to average out small variations in error across independent runs. Reference images were rendered with 32,768 samples per pixel (spp).

For the first two additional environment maps, SZ also reduces error compared to Sobol sampling; *Sky* has slightly higher error with SZ than with Sobol samples, though error for both is low. In general, the greatest improvement seems to come with the most complex environment maps. Given both a complex BSDF and complex illumination, sampling their full 4D product well is important for accurate rendering. We hypothesize that the scenes with more complex environments see more improvement thanks to SZ providing a $(0, 4)$ -sequence for those dimensions.

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MRSE visualizations for the entire images with both samplers are shown in Figure S2. We can see that the error reduction is greatest with the complex glossy specular BSDF of the car's paint, though for some scenes there is also some benefit for the near-diffuse ground plane and background.

Error at different numbers of samples per pixel is reported in Tables S1 and S2. SZ has lower MRSE than Sobol over a range of power-of-4 sampling rates and also has lower FLIP error at low sampling rates. At higher sampling rates, both have very low FLIP error. It is interesting to note that the relative performance of SZ and Sobol can be quite different at different sampling rates, even for the same scene. This is somewhat unexpected, as we generally expect MRSE to decrease at a constant rate with increasing sampling rate. We also see that with *Sky*, SZ does extremely well at 16 and 256spp—both even powers of 4 samples. At those rates, SZ has nearly 3× lower error than Sobol sampling.

We have also measured performance of our sampler on a 32-core AMD 3970X CPU and an NVIDIA A6000 GPU. We find that there is no meaningful performance difference when using the SobolSampler or our SZSampler; the time spent generating sample points is less than 3% of total rendering time.

REFERENCES

- Lukáš Hošek and Alexander Wilkie. 2012. An Analytic Model for Full Spectral Sky-Dome Radiance. *ACM Transactions on Graphics* 31, 4, Article 95 (July 2012), 9 pages. <https://doi.org/10.1145/2185520.2185591>
- Lukáš Hošek and Alexander Wilkie. 2013. Adding a Solar-Radiance Function to the Hošek-Wilkie Skylight Model. *IEEE Computer Graphics and Applications* 33, 3 (2013), 44–52. <https://doi.org/10.1109/MCG.2013.18>



Fig. S1. Comparisons of images rendered using the Sobol sequence and using our SZ sequence. All images are rendered with 64spp with direct lighting only, using four varied environment maps. Crops show representative results; full images are available in our supplemental material. See Figure S2 for the specification of the heatmap used to visualize MRSE error.

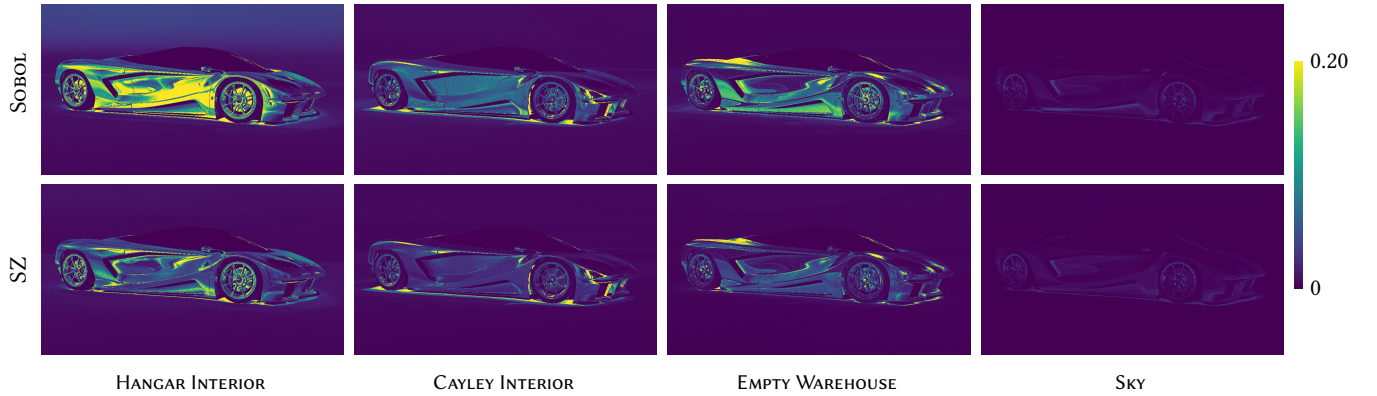


Fig. S2. Visualizations of MRSE for the four scenes shown in Figure S2, generated with test images rendered at 64spp. The benefit from SZ points is generally small in the relatively diffuse ground-plane and background; the largest reductions in error are on the car body, which has a complex BSDF.

spp	Hangar Interior			Cayley Interior			Empty Warehouse			Sky		
	Sobol	SZ	Ratio	Sobol	SZ	Ratio	Sobol	SZ	Ratio	Sobol	SZ	Ratio
16	0.128	0.111	1.15×	0.0856	0.0697	1.23×	0.111	0.0803	1.38×	0.0329	0.0111	2.96×
64	0.0372	0.0193	1.93×	0.0182	0.0122	1.49×	0.0213	0.0133	1.60×	0.00147	0.00164	0.897×
256	0.00383	0.00393	0.977×	0.0029	0.00216	1.34×	0.00297	0.00227	1.31×	0.000412	0.000152	2.71×
1024	0.000945	0.000838	1.13×	0.000914	0.000519	1.76×	0.000702	0.000645	1.09×	0.000046	0.000054	0.852×

Table S1. MRSE the four *Sportscar* scenes at various power-of-4 numbers of pixel samples. For most scenes and most sampling rates, MRSE is lower with the SZ sampler than with the Sobol sampler. For *Sky* SZ sampling yields much lower error at 16 and 256spp but slightly higher at 64 and 1024spp.

spp	Hangar Interior			Cayley Interior			Empty Warehouse			Sky		
	Sobol	SZ	Diff.	Sobol	SZ	Diff.	Sobol	SZ	Diff.	Sobol	SZ	Diff.
16	0.101	0.0986	0.00245	0.0852	0.0795	0.00573	0.0762	0.0712	0.00497	0.0413	0.0297	0.0117
64	0.0497	0.0473	0.00237	0.0379	0.0348	0.00314	0.0351	0.0323	0.00285	0.0126	0.0122	0.000409
256	0.024	0.0255	-0.00154	0.0192	0.0194	-0.000228	0.017	0.0167	0.000275	0.00606	0.00622	-0.000159
1024	0.0137	0.0151	-0.00149	0.0111	0.0119	-0.000744	0.00934	0.0107	-0.00132	0.0029	0.00482	-0.00193

Table S2. FLIP error for the test scenes at various power-of-4 numbers of pixel samples. FLIP error is consistently lower with the SZ sampler at 16 and 64 samples per pixel; at 256 and 1024 samples per pixel, the image is nearly converged and FLIP reports nearly equal error for both samplers.